

112 年特種考試地方政府公務人員考試試題

等 別：三等考試

類 科：交通技術

科 目：交通統計

吳迪老師

一、試解釋何謂統計量之不偏性及有效性。(10 分)

1. 《考題難易》★★

2. 《破題關鍵》：考點估計的優良性質，基本題。

【擬答】

(一)不偏性 (Unbiased)

1. 意義：

以樣本統計量 $\hat{\theta}$ 估計母數 θ 時，

(1)若 $E(\hat{\theta}) = \theta$ ，則 $\hat{\theta}$ 稱為 θ 的不偏估計式 (Unbiased Estimator)。

(2)若 $E(\hat{\theta}) \neq \theta$ ，則 $\hat{\theta}$ 稱為 θ 的偏誤估計式 (Biased Estimator)。

其中偏誤 $\text{Bias} = E(\hat{\theta}) - \theta$

(二)有效性 (Efficiency)

1. 意義：

對任何一個母數不一定只有一個不偏估計式。而在樣本數 n 固定下，則變異數愈小的不偏式即愈有效。

2. 相對有效 (Relatively Efficiency)：

設 $(X_1, X_2, \dots, X_n) \stackrel{\text{i.i.d.}}{\sim} f(x; \theta)$

(1)若 $\hat{\theta}_1$ 、 $\hat{\theta}_2$ 為 θ 之二個不偏估計式，且 $\text{Var}(\hat{\theta}_1) < \text{Var}(\hat{\theta}_2)$ ，則以 $\hat{\theta}_1$ 估計 θ 比以 $\hat{\theta}_2$ 估計 θ 相對的有效。

(2)若 $\hat{\theta}_1$ 、 $\hat{\theta}_2$ 為 θ 之二個任意估計式，且 $\text{MSE}(\hat{\theta}_1) < \text{MSE}(\hat{\theta}_2)$ ，則以 $\hat{\theta}_1$ 估計 θ 比以 $\hat{\theta}_2$ 估計 θ 相對的有效。

其中 $\text{MSE}(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$

$= \text{Var}(\hat{\theta}) + \text{Bias}^2$ ，即 $\text{MSE}(\hat{\theta})$ 愈小，表示偏誤 Bias 愈小，則愈有效。

3. 絕對有效 (Absolutely Efficiency)：

(1)在所有不偏估計式中，若 $\hat{\theta}$ 的變異數最小，則 $\hat{\theta}$ 稱為 θ 的最小變異不偏估計式 (Minimum Variance Unbiased Estimation)，簡稱 MVUE。

公職王歷屆試題 (112 地方政府特考)

二、根據交通部統計處某年度之計程車營運狀況調查，某計程車每天所耗用之油料費用為一常態分配，其平均值為 382 元，標準差為 112 元。(每小題 5 分，共 20 分)

(一)當隨機抽取一輛計程車，其每天耗用之油料成本超過 450 元之機率為何？

(二)續題(一)，請說明題(一)中所估算之機率之意義。

(三)某家計程車行有 6 輛計程車，則 6 輛計程車每天平均的油料費用為何？其平均值超過 450 元的機率為何？

(四)續題(三)，該車行每天油料費用少於 2,200 元的機率為何？

1. 《考題難易》★★

2. 《破題關鍵》：考常態分配的機率,基本題

【擬答】

$$(一) X \sim N(\mu = 382, \sigma^2 = 112^2)$$

$$P(X > 450) = P\left(Z > \frac{450 - 382}{112}\right) = P(Z > 0.61) = 0.2709$$

(二)某年度的計程車中有 27.09%每天耗用的油料成本會超過 450 元。

$$(三) 1. E(\bar{X}) = \mu = 382 \text{ 元}$$

$$2. P(\bar{X} > 450) = P\left(Z > \frac{450 - 382}{\frac{112}{\sqrt{6}}}\right) = P(Z > 1.49) = 0.0681$$

$$(四) \text{設 } Y = X_1 + X_2 + \cdots + X_6$$

$$\Rightarrow E(Y) = 382 \times 6 = 2292$$

$$\text{Var}(Y) = 112^2 \times 6 = 75264$$

$$\Rightarrow P(Y < 2200) = P\left(Z < \frac{2200 - 2292}{\sqrt{75264}}\right) = P(Z < -0.34) = 0.3669$$

三、假設目前所屬單位所使用之號誌設備為 A 廠牌，而 A 牌號誌設備的平均使用年限為 8 年，標準差為 1.4 年。所屬單位目前正考慮採購宣稱平均使用年限可較目前 A 牌設備長 1.5 年之 B 牌設備。A、B 兩種品牌的號誌設備廠商各提供 64 個樣本。B 牌號誌設備的平均使用年限為 9 年，標準差為 1.9 年。試問 B 牌號誌設備平均使用年限是否比 A 牌號誌設備長 1.5 年？試闡述您是否會接受 B 牌設備廠商之說法？(所需統計表請見附件)(20 分)

1. 《考題難易》★★

2. 《破題關鍵》：考二獨立母體平均數的檢定,基本題

【擬答】

$$\begin{cases} H_0: \mu_B - \mu_A \geq 1.5 \\ H_1: \mu_B - \mu_A < 1.5 \end{cases}$$

$N_A - N_B = 64 \geq 30$ ，由 C.L.T，利用 Z 檢定。

設 $\alpha = 0.05$

拒絕域 $C = \{Z | Z < -1.645\}$

$$Z = \frac{(\bar{X}_B - \bar{X}_A) - (M_B - M_A)}{\sqrt{\frac{S_A^2}{N_A} + \frac{S_B^2}{N_B}}} = \frac{(9 - 8) - 1.5}{\sqrt{\frac{1.4^2}{64} + \frac{1.9^2}{64}}} = -1.69 \in C \Rightarrow \text{ReHo}$$

結論：B 牌號誌設備平均使用年限不會比 A 牌設備長 1.5 年。所以不接受 B 牌設備廠商之說法。

四、某交通工程師欲估計某商場開幕後，對鄰近路口的服務水準影響為何，以下為該商場鄰近 9 處路口的停等延滯資料。試問事前事後停等延滯時間差的 95%信賴區間為何？該商場是否顯著增加鄰近路口的停等延滯？（所需統計表請見附件）(20 分)

路口	1	2	3	4	5	6	7	8	9
事前停等延滯 (秒)	31.5	17.2	22.2	45.8	33.3	18.8	26.7	50.6	31.4
事後停等延滯 (秒)	35.6	19.2	25.0	50.3	37.8	21.0	32.3	54.5	36.9

1.《考題難易》★★

2.《破題關鍵》：考二相關母體平均數的信賴區間,常考題

【擬答】

路口	1	2	3	4	5	6	7	8	9
事前(X)	31.5	17.2	22.2	45.8	33.3	18.8	26.7	50.6	31.4
事後(Y)	35.6	19.2	25.0	50.3	37.8	21.0	32.3	54.5	36.9
D=Y-X	4.1	2	2.8	4.5	4.5	2.2	5.6	3.9	5.5

$$\Sigma D = 35.1, \Sigma D^2 = 150.81$$

$$\bar{D} = 3.9, S_D = 1.32$$

設母體為常態，且為相依樣本，利用 t 分配，則 $M_Y - M_X$ 信賴度 95% 之信賴區間

$$\left(\bar{D} - t_{0.025}(8) \frac{S_D}{\sqrt{N}}, \bar{D} + t_{0.025}(8) \frac{S_D}{\sqrt{N}} \right)$$

$$\Rightarrow \left(3.9 - 2.306 \times \frac{1.32}{\sqrt{9}}, 3.9 + 2.306 \times \frac{1.32}{\sqrt{9}} \right) \Rightarrow (2.89, 4.91)$$

因為信賴區間大於 0，即 $M_Y > M_X$ ，所以該商場會顯著增加鄰近路口的停等延滯。

志光·學儒·保成

五大學習方式 上課超便利



現場面授
名師現場面對面
即時互動解答疑惑



直播教學
即時登入直播跟課
掌握進度免等待



視訊課程
手機APP預約上課
輔導期間 無限重複看課



WIFI看課
專屬WIFI教室
讓你學習時間更彈性



在家學習
使用在家補課點數
即可在家複習上課
(以老師授權科目為主)

五、某縣市政府想分析人口數(十萬人)(X)與延車公里的(百萬公里)(Y)之間的關係。根

據過去 10 年的資料,得如下的資料: $\sum X = 28$,

$$\sum X^2 = 303.4, \sum Y = 75, \sum Y^2 = 598.5, \sum XY = 237. \text{(每小題 10 分, 共 30 分)}$$

(一)試求迴歸直線 $\hat{Y} = \hat{\alpha} + \hat{\beta} X$ 。

(二)試檢定「人口越多,延車公里也越高」的假設(顯著水準為 5%)。

(三)請闡述上述的關係是否為因果關係?

1.《考題難易》★★

2.《破題關鍵》:考簡單迴歸與相關,基本題

【擬答】

$$(一)\hat{Y} = \hat{\alpha} + \hat{\beta}X$$

$$1. \hat{\beta} = \frac{SS_{XY}}{SS_X} = \frac{237 - \frac{28 \times 75}{10}}{303.4 - \frac{28^2}{10}} = 0.12$$

$$2. \hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X} = \frac{75}{10} - 0.12 \times \frac{28}{10} = 7.164 \Rightarrow \hat{Y} = 7.164 + 0.12X$$

$$(二)r = \frac{S_{XY}}{S_X S_Y} = \frac{237 - \frac{28 \times 75}{10}}{\sqrt{303.4 - \frac{28^2}{10}} \sqrt{598.5 - \frac{75^2}{10}}} = 0.3$$

$$\begin{cases} H_0: \rho \leq 0 \\ H_1: \rho > 0 \end{cases}$$

$$\alpha = 0.05$$

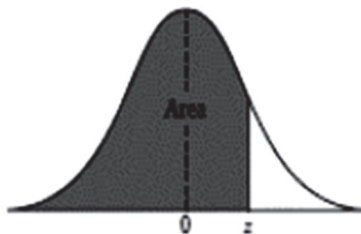
$$\text{拒絕域 } C = \{t | t > 2.306 \text{ 或 } t < -2.306\}$$

$$\text{檢定統計量 } t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.3\sqrt{10-2}}{\sqrt{1-0.3^2}} = 0.889 \notin C \Rightarrow \text{not ReHo}$$

結論:沒有證據顯示人口越多,延車公里數也越高。

(三)上述的關係非因果關係,只是探討 X, Y 是否有線性關係。

附件



Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

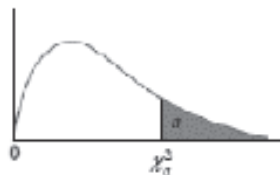
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998



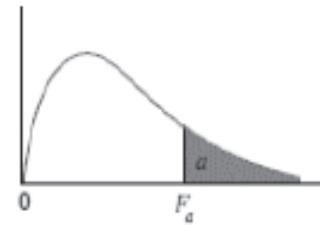


Critical Values of t	df	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	df
	1	3.078	6.314	12.706	31.821	63.657	1
	2	1.886	2.920	4.303	6.965	9.925	2
	3	1.638	2.353	3.182	4.541	5.841	3
	4	1.533	2.132	2.776	3.747	4.604	4
	5	1.476	2.015	2.571	3.365	4.032	5
	6	1.440	1.943	2.447	3.143	3.707	6
	7	1.415	1.895	2.365	2.998	3.499	7
	8	1.397	1.860	2.306	2.896	3.355	8
	9	1.383	1.833	2.262	2.821	3.250	9
	10	1.372	1.812	2.228	2.764	3.169	10
	11	1.363	1.796	2.201	2.718	3.106	11
	12	1.356	1.782	2.179	2.681	3.055	12
	13	1.350	1.771	2.160	2.650	3.012	13
	14	1.345	1.761	2.145	2.624	2.977	14
	15	1.341	1.753	2.131	2.602	2.947	15
	16	1.337	1.746	2.120	2.583	2.921	16
	17	1.333	1.740	2.110	2.567	2.898	17
	18	1.330	1.734	2.101	2.552	2.878	18
	19	1.328	1.729	2.093	2.539	2.861	19
	20	1.325	1.725	2.086	2.528	2.845	20
	21	1.323	1.721	2.080	2.518	2.831	21
	22	1.321	1.717	2.074	2.508	2.819	22
	23	1.319	1.714	2.069	2.500	2.807	23
	24	1.318	1.711	2.064	2.492	2.797	24
	25	1.316	1.708	2.060	2.485	2.787	25
	26	1.315	1.706	2.056	2.479	2.779	26
	27	1.314	1.703	2.052	2.473	2.771	27
	28	1.313	1.701	2.048	2.467	2.763	28
	29	1.311	1.699	2.045	2.462	2.756	29
	∞	1.282	1.645	1.960	2.326	2.576	∞

Source: From "Table of Percentage Points of the t -Distribution," *Biometrika* 32 (1941):300. Reproduced by permission of the *Biometrika* Trustees.



Critical Values of Chi-Square	χ^2_{100}	χ^2_{050}	χ^2_{025}	χ^2_{010}	χ^2_{005}	df
	2.70554	3.84146	5.02389	6.63490	7.87944	1
	4.60517	5.99147	7.37776	9.21034	10.5966	2
	6.25139	7.81473	9.34840	11.3449	12.8381	3
	7.77944	9.48773	11.1433	13.2767	14.8602	4
	9.23635	11.0705	12.8325	15.0863	16.7496	5
	10.6446	12.5916	14.4494	16.8119	18.5476	6
	12.0170	14.0671	16.0128	18.4753	20.2777	7
	13.3616	15.5073	17.5346	20.0902	21.9550	8
	14.6837	16.9190	19.0228	21.6660	23.5893	9
	15.9871	18.3070	20.4831	23.2093	25.1882	10
	17.2750	19.6751	21.9200	24.7250	26.7569	11
	18.5494	21.0261	23.3367	26.2170	28.2995	12
	19.8119	22.3621	24.7356	27.6883	29.8194	13
	21.0642	23.6848	26.1190	29.1413	31.3193	14
	22.3072	24.9958	27.4884	30.5779	32.8013	15
	23.5418	26.2962	28.8485	31.9999	34.2672	16
	24.7690	27.8571	30.1910	33.4087	35.7185	17
	25.9894	28.8693	31.5264	34.8053	37.1564	18
	27.2036	30.1435	32.8523	36.1908	38.5822	19
	28.4120	31.4104	34.1696	37.5662	39.9968	20
	29.6151	32.6705	35.4789	38.9321	41.4010	21
	30.8133	33.9244	36.7807	40.2894	42.7956	22
	32.0069	35.1725	38.0757	41.6384	44.1813	23
	33.1963	36.4151	39.3641	42.9798	45.5585	24
	34.3816	37.6525	40.6465	44.3141	46.9278	25
	35.5631	38.8852	41.9232	45.6417	48.2899	26
	36.7412	40.1133	43.1944	46.9630	49.6449	27
	37.9159	41.3372	44.4607	48.2782	50.9933	28
	39.0875	42.5569	45.7222	49.5879	52.3356	29
	40.2560	43.7729	46.9792	50.8922	53.6720	30
	51.8050	55.7585	59.3417	63.6907	66.7659	40
	63.1671	67.5048	71.4202	76.1539	79.4900	50
	74.3970	79.0819	83.2976	88.3794	91.9517	60
	85.5271	90.5312	95.0231	100.425	104.215	70
	96.5782	101.879	106.629	112.329	116.321	80
	107.565	113.145	118.136	124.116	128.299	90
	118.498	124.342	129.561	135.807	140.169	100

Percentage Points of the F Distribution

df_2	α	df_1								
		1	2	3	4	5	6	7	8	9
1	.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	.050	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
	.025	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3
	.010	4052	4999.5	5403	5625	5764	5859	5928	5982	6022
	.005	16211	20000	21615	22500	23056	23437	23715	23925	24091
2	.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	.005	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4
3	.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	.010	34.12	30.82	29.46	28.71	28.24	27.91	27.64	27.49	27.35
	.005	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88
4	.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	.005	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14
5	.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	.005	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77
6	.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.005	18.63	14.54	12.92	12.03	11.46	11.07	10.79	10.57	10.39
7	.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.005	16.24	12.40	10.88	10.05	9.52	9.16	8.89	8.68	8.51
8	.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	.005	14.69	11.04	9.60	8.81	8.30	7.95	7.69	7.50	7.34
9	.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	.005	13.61	10.11	8.72	7.96	7.47	7.13	6.88	6.69	6.54

df_1										a	df_2
10	12	15	20	24	30	40	60	120	∞		
60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33	.100	1
241.9	243.9	245.9	248.0	249.1	250.1	251.2	252.2	253.3	254.3	.050	
968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018	.025	
6056	6106	6157	6209	6235	6261	6287	6313	6339	6366	.010	
24224	24426	24630	24836	24940	25044	25148	25253	25359	25465	.005	
9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49	.100	2
19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	.050	
39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50	.025	
99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	.010	
199.4	199.4	199.4	199.4	199.5	199.5	199.5	199.5	199.5	199.5	.005	
5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13	.100	3
8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	.050	
14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90	.025	
27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13	.010	
43.69	43.39	43.08	42.78	42.62	42.47	42.31	42.15	41.99	41.83	.005	
3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76	.100	4
5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	.050	
8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26	.025	
14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46	.010	
20.97	20.70	20.44	20.17	20.03	19.89	19.75	19.61	19.47	19.32	.005	
3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10	.100	5
4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36	.050	
6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02	.025	
10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02	.010	
13.62	13.38	13.15	12.90	12.78	12.66	12.53	12.40	12.27	12.14	.005	
2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72	.100	6
4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	.050	
5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85	.025	
7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88	.010	
10.25	10.03	9.81	9.59	9.47	9.36	9.24	9.12	9.00	8.88	.005	
2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47	.100	7
3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	.050	
4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14	.025	
6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65	.010	
8.38	8.18	7.97	7.75	7.65	7.53	7.42	7.31	7.19	7.08	.005	
2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29	.100	8
3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	.050	
4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67	.025	
5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86	.010	
7.21	7.01	6.81	6.61	6.50	6.40	6.29	6.18	6.06	5.95	.005	
2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16	.100	9
3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	.050	
3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33	.025	
5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31	.010	
6.42	6.23	6.03	5.83	5.73	5.62	5.52	5.41	5.30	5.19	.005	

(continued)

df ₂	df ₁									
	a	1	2	3	4	5	6	7	8	9
10	.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.005	12.83	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97
11	.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
	.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	.005	12.23	8.91	7.60	6.88	6.42	6.10	5.86	5.68	5.54
12	.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	.005	11.75	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
13	.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
	.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	.005	11.37	8.19	6.93	6.23	5.79	5.48	5.25	5.08	4.94
14	.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
	.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	.005	11.06	7.92	6.68	6.00	5.56	5.26	5.03	4.86	4.72
15	.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.005	10.80	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54
16	.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
	.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	.005	10.58	7.51	6.30	5.64	5.21	4.91	4.69	4.52	4.38
17	.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
	.010	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68
	.005	10.38	7.35	6.16	5.50	5.07	4.78	4.56	4.39	4.25
18	.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
	.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
	.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	.005	10.22	7.21	6.03	5.37	4.96	4.66	4.44	4.28	4.14
19	.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
	.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
	.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	.005	10.07	7.09	5.92	5.27	4.85	4.56	4.34	4.18	4.04
20	.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	.005	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96

(continued)

df ₁										a	df ₂
10	12	15	20	24	30	40	60	120	∞		
2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06	.100	10
2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	.050	
3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08	.025	
4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91	.010	
5.85	5.66	5.47	5.27	5.17	5.07	4.97	4.86	4.75	4.64	.005	
2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97	.100	11
2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	.050	
3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88	.025	
4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60	.010	
5.42	5.24	5.05	4.86	4.76	4.65	4.55	4.44	4.34	4.23	.005	
2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90	.100	12
2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	.050	
3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72	.025	
4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36	.010	
5.09	4.91	4.72	4.53	4.43	4.33	4.23	4.12	4.01	3.90	.005	
2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85	.100	13
2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	.050	
3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60	.025	
4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17	.010	
4.82	4.64	4.46	4.27	4.17	4.07	3.97	3.87	3.76	3.65	.005	
2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80	.100	14
2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	.050	
3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49	.025	
3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00	.010	
4.60	4.43	4.25	4.06	3.96	3.86	3.76	3.66	3.55	3.44	.005	
2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76	.100	15
2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	.050	
3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40	.025	
3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87	.010	
4.42	4.25	4.07	3.88	3.79	3.69	3.58	3.48	3.37	3.26	.005	
2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72	.100	16
2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	.050	
2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32	.025	
3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75	.010	
4.27	4.10	3.92	3.73	3.64	3.54	3.44	3.33	3.22	3.11	.005	
2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69	.100	17
2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	.050	
2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25	.025	
3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65	.010	
4.14	3.97	3.79	3.61	3.51	3.41	3.31	3.21	3.10	2.98	.005	
1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66	.100	18
2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	.050	
2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19	.025	
3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57	.010	
4.03	3.86	3.68	3.50	3.40	3.30	3.20	3.10	2.99	2.87	.005	
1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63	.100	19
2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	.050	
2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13	.025	
3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49	.010	
3.93	3.76	3.59	3.40	3.31	3.21	3.11	3.00	2.89	2.78	.005	
1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61	.100	20
2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	.050	
2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09	.025	
3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42	.010	
3.85	3.68	3.50	3.32	3.22	3.12	3.02	2.92	2.81	2.69	.005	